ABSTRACT

In dealing with subsurface engineering problems concerning rock mechanics, the determination of in-situ physical properties of rocks has assumed great importance. In recent years, various techniques have been applied in correlating the log-derived parameters with mechanical behavior of the rocks. These techniques have found applications in the petroleum, mining and construction industries.

REFERENCES AND ILLUSTRATIONS AT END OF PAPER.

INTRODUCTION

Specific in-situ elastic properties of the formations must be estimated for resolving problems of rock mechanics. Mechanical behavior of porous rocks depends upon a number of factors: (1) elastic modulus values, (2) variation in elastic moduli with stress conditions, (3) overburden weight gradients, (4) stresses caused by geological conditions, (5) strength of cementation between grains, (6) fluid pressures, (7) fluid saturations, (8) rates of flow, and (9) fracture pressure gradients. Elastic parameters are the key to predicting the mechanical behavior of rocks. Theoretical relations based upon the theory of elasticity are generally found to be valid for the calculation of elastic parameters in subsurface formations. Quantitative determination of rock elastic constants from well log measurements has recently assumed importance. This is because of improved technology of these measurements and greater computing capability available for field processing of borehole logging data.

In terms of well logging parameters and in practical units, the elastic constants for the rock formations are expressed as:

\[ Y = \left( \frac{\rho_b}{\Delta t_i} \right) \left( \frac{3\Delta t_i^2 - 4\Delta t_e^2}{\Delta t_i^2 - \Delta t_e^2} \right) \times 1.34 \times 10^{10} \text{ psi} \]  

\[ B = \rho_b \left( \frac{3\Delta t_i^2 - 4\Delta t_e^2}{3\Delta t_i^2 \times \Delta t_e^2} \right) \times 1.34 \times 10^{10} \text{ psi} \]  

\[ S = \left( \rho_b/\Delta t_i \right) \times 1.34 \times 10^{10} \text{ psi} \]  

\[ P = 0.5 \left( \frac{\Delta t_i^2 - 2\Delta t_e^2}{\Delta t_i^2 - \Delta t_e^2} \right) \]  

With sedimentary rocks, a difference is found to exist between the elastic moduli measured in a conventional testing machine and those determined by acoustic (dynamic) methods. The difference has been attributed to the slipping and frictional effects occurring at the contacts between grains during acoustic measurements. This is explained through a set of equations by Gassman. The dynamic and static bulk modulus values have recently been correlated from data measured on sandstones as shown in Figure 1.

The correlation is noted to be a function of porosity. Dynamic values of elastic moduli of porous rocks are observed to be consistently higher than static values. However, the comparison between elastic constants determined by full-wave train acoustic logs under in-situ conditions show good agreement with dynamic laboratory measurements on rock samples under simulated formation pressures. From the practical standpoint of predicting rock mechanical behavior, dynamic measurements from well logs are considered more valid.
WELL LOGGING METHODS

Measurements of densities and acoustic wave velocities are essential for the analysis of rock mechanical properties. The Dresser Atlas Compensated Densilog® service provides in-situ bulk density data of the formations penetrated by the borehole, minimizing the adverse effects of mud cake against porous intervals and borehole variations. However, acoustic measurements primarily define the rock mechanical behavior, while the effect of density is too little. Measurement of the compressional wave travel times of the rock formations, largely balancing the effects of irregular and deviated boreholes, is obtained from the Dresser Atlas BHC Acoustilog®. The Acoustilog provides excellent porosity resolutions in deep well compacted sediments, but in shallower unconsolidated formations the specific transit times are influenced by other geological factors as well.

Because shear arrivals are often out of phase with the compressional wave arrivals, they cause interference on the total wave train. They are normally stronger in amplitude, except when they are attenuated by traversing across fractures.

Different types of acoustic logging systems have been used for the measurement of shear wave velocity in rock formations. The first system, called the Sidewall Acoustilog, has a shorter spacing (nine inches from transmitter to first receiver and six inches from first to second receiver) than the conventional Acoustilog. The acoustic recording pad is pressed against the borehole wall. Many of the problems associated with borehole fluid are thus avoided. The system has been helpful in the detection of fractures which are nearly horizontal because of the inability of shear energy to travel through fractured zones. The Sidewall Acoustilog provides continuous recordings of both P- and S-wave specific transit times sequentially. However, problems may be encountered in S-wave identification in soft rocks. Besides, there is usually considerable uncertainty in identification of shear arrivals because of P-wave interference on short-spaced tools. With conventional tools, the electronic amplifiers are adjusted to accentuate the amplitude of P-wave arrival. They reach saturation through S-wave arrival. If amplification is reduced to avoid saturation, the interference due to late P-wave arrival precludes the automatic detection of S-wave arrival. Therefore, although the conventional Acoustilog spacings allow adequate time for distinguishing P- and S-waves, the latter is highly attenuated in amplitude due to increased distance of travel, except in certain hard rocks such as dense carbonates. Acoustic P- and S-wave transit times can be successfully measured in compact limestone formations by a standard Acoustilog. Such recordings by a Sidewall Acoustilog have a greater lithology range of applicability (Fig. 2).

A long spaced BHC Acoustilog has also been used for the measurement of shear wave specific transit times. A spacing of 7 feet between each transmitter and nearer receiver facilitates the differentiation between P- and S-wave arrivals. In this system, high energy acoustic pulses are employed because attenuation due to the large distance of travel increases. The other acoustic logging system used for the determination of shear wave velocity is the Variable Density/Sigature Log. The equipment includes an oscilloscope, which records the acoustic wave train in rapid succession after each pulse from the transmitter. The film moves in synchronization with the borehole instrument. The recording can be in the form of brightness of the spot varying according to the acoustic wave amplitude (Variable Density) or in the form of curves showing varying amplitude with time (Signature Log). The entire wave train arriving at the receiver is permanently recorded, making it possible to study the complete log in detail at any time. On the Variable Density Log, because the brightness at any instant depends upon the amplitude of received wave, the wave train is presented in the form of a succession of varying shades of brightness. The darkest or brightest bands correspond to maximum amplitudes within the wave train in positive or negative directions. Thus the contrast between successive dark and bright bands is proportional to signal strength at that instant. The Signature Log presents the same information in a standard wave form as on a seismogram (Fig. 3). Variations are indicated by the deflection of the trace from horizontal.

A better contrast between the bands, where a change of slope of the band is also usually observed, indicates the arrival of shear wave. Spots 1 and 2 in Figure 3 signify the arrivals of compressional and shear waves respectively.

MECHANICAL BEHAVIOR OF FRIABLE SANDS

By means of elastic constants, certain in-situ mechanical properties of rock formations can be determined. With friable sands, this is important in order to avoid sand production. Once a well starts making sand, it is difficult and expensive to implement effective sand control measures. Accuracy of determination of in-situ mechanical properties from well logs is higher than from laboratory tests on core samples. Removal of overburden pressure during core recovery results in expansion, which can cause breaking of weak bonds between the grains. In such cases, the strength due to cementation would not be detected during tests at the surface.

Production of sand along with oil and gas is a formidable problem in many younger, unconsolidated
formations. The object of determining formation strength from well logs is to know whether the formation is strong enough to produce without making sand at high flow rates. If the formation is weak, it is beneficial to determine the optimum production rate which can be sustained without any form of sand production or otherwise required sand control. Increased rates are often accompanied by sanding problems because the sand grains are torn away from the rock matrix with fluid movement into the wellbore under large pressure differential. Ironically, strong sands which would not present any danger of sand production frequently have lower permeabilities and are therefore incapable of very high flow rates even with large pressure drawdowns. On the other hand, weak sands usually possess higher permeabilities but can cause sanding problems, even at moderate rates of production with small pressure drawdowns. A knowledge of mechanical properties of these friable sand reservoirs provides a basis for obtaining economically attractive and sustainable production rates.

A number of other factors must be considered in order to predict mechanical behavior of friable sands. Formation of stable load-carrying arches around the perforation cavities can build resistance to the destructive forces due to fluid movement under pressure gradient.\(^{(9)}\) If there is sufficient cohesion between the sand grains, these arches can withstand substantial stress. The cohesive forces are due to mineral cementation between grains as well as the capillary forces caused by interstitial water in hydrocarbon-producing formations. Laboratory tests\(^{(10)}\) have indicated that the condition of irreducible water saturation in a water wet sandstone contributes to cohesion. The sand grains adhere to each other because of these capillary forces. At high water saturation, fluid interfaces are away from the sand grains, thus weakening these forces. This is supported by field observations that high water cut is often accompanied by sand production in a number of wells. Stable arches can be formed in uncemented granular material where cohesion is provided by capillary forces only.

Strength of sand formations can be estimated by deriving elastic constants through the use of well log data. There is considerable evidence that a good correlation exists between intrinsic strength of rock formation and the dynamic elastic constants. It has been shown\(^{(9)}\) that shear modulus is the single most important elastic parameter in comparing strength of different formations. Bulk modulus, which represents resistance to compressibility, is fairly constant for friable sands over the usual range of load conditions. Very shallow, uncompacted sands have little or no shear strength. However, they possess resistance to change in bulk volume not much lower than that of deeper formations. Under certain stress conditions, sands exhibit significant shear strength, causing the adjoining particles to adhere to each other.

A combined modulus of strength\(^{(9,11)}\), \(K\), has been defined as,

\[
K = B + \frac{4}{3} S
\]

This can be expressed in well logging terms as,

\[
K = \frac{p_s}{\Delta t} \times 1.34 \times 10^{10} \text{ psi}
\]

It should be realized that the combined modulus may not define the strength characteristics of rock formations under all conditions. Acoustic and density logs measure average values of \(\Delta t\) and \(p_s\) along the borehole wall. Minute variations in the rock structure, which may not affect these logs, can alter the intrinsic strength of the formation. However, there is enough evidence from published work that the combined modulus of strength compares favorably with known conditions of formation strength.

A combined modulus of \(1.5 \times 10^6\) psi or less usually indicates sand production. A value between \(1.5 \times 10^6\) psi and \(3 \times 10^6\) psi generally represents a condition where sand will not be produced below a specific flow rate. This condition is called the rate sensitive zone. A value of \(K\) greater than \(3 \times 10^6\) psi indicates the safe zone, i.e., reservoir fluids may be produced into the wellbore at any desirable rate.

Another approach\(^{(8)}\) to the estimation of relative formation strength has been to calculate the ratio of shear modulus (\(S\)) to compressibility (\(C\)). As compressibility is the reciprocal of Bulk Modulus (\(B\)), this ratio is the same as the product of Shear and Bulk Modulii. This parameter has been correlated with production results in a number of tertiary basins within and outside the United States. It is observed that for production without sand problems, \(S/C\) or \(S \times B \geq 0.8 \times 10^{12}\) psi\(^2\).

In computing this parameter, a knowledge of \(p_s\) and \(\Delta t\), are not enough and it is necessary to measure \(\Delta t\) (specific acoustic time for shear wave) also. This can be obtained by running a Sidewall Acoustilog\(^a\) or a Variable Density/Signature Log. However, as mentioned before, the measurement of \(\Delta t\) in friable sediments is very elusive. The shear wave is highly attenuated and usually difficult to distinguish from the interfering fluid arrivals.

An alternate approach has been to determine \(\Delta t\), indirectly by a correlation of another elastic constant, Poisson's Ratio, to the shaliness index \((q)\) obtained from acoustic and density logs.\(^{(12)}\) The shaliness index is expressed as:
Poisson’s Ratio has been related to the shaliness index in the following manner:[12]

\[ P = 0.125 q + 0.27 \]  

Therefore, when \( \Delta t \) is not available, it is possible to compute all the elastic constants by deriving Poisson’s Ratio through empirical correlation with shaliness index in sands.

The threshold value of 0.8 \( \times 10^{12} \) psi\(^2\) for the S/C ratio has been found to apply well in the case of tertiary formations throughout the areas where these studies have been reported. These include Gulf Coast, Texas and Louisiana offshore, California and some areas outside of the United States. This ratio may be as low as 0.3 \( \times 10^{12} \) psi\(^2\) in very loose sands and may exceed 6.0 \( \times 10^{12} \) psi\(^2\) for an extremely strong formation. No case has been reported in which sanding has occurred when the S/C ratio was greater than the threshold value. In addition, sand control was found to be necessary whenever the S/C ratio was equal to or less than 0.7 \( \times 10^{12} \) psi\(^2\).

Both the combined modulus of strength and the S/C ratio are essentially indicative of relative strength of formations. These have been particularly useful where well logs indicated the formations to be competent, although sanding problems had been anticipated. In such cases, higher production rates were achieved without using sand control.

**LITHOLOGY AND ELASTIC PROPERTIES**

The effect of rock lithology on elastic wave velocities was studied by Pickett\(^{[13]}\) by taking laboratory measurements from core samples. A definite correlation was found to exist between the lithology of sedimentary rocks and the ratio of reciprocals of shear and compressional wave velocities. The relationship between velocities of the two modes of elastic wave propagation therefore provides a means of lithology identification from well logs alone and is found to be relatively independent of variations in effective stress and porosity. Typical values of the ratio of specific acoustic time for shear wave and specific acoustic time for compressional wave \( (\Delta t / \Delta t_c) \) in different lithologies are:

<table>
<thead>
<tr>
<th>Lithology</th>
<th>( \Delta t / \Delta t_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sandstones</td>
<td>1.60 to 1.75</td>
</tr>
<tr>
<td>Limy Sands</td>
<td>1.70</td>
</tr>
<tr>
<td>Dolomites</td>
<td>1.80</td>
</tr>
<tr>
<td>Limestones</td>
<td>1.90</td>
</tr>
</tbody>
</table>

Figure 4 shows plots of lines of equal ratio \( (\Delta t / \Delta t_c) \) indicating that dolomites and limestones are characterized by different but constant ratio values. Sandstones, on the other hand, exhibit variations in the range from 1.60 for low-porosity sandstones to about 1.75 for high-porosity sands. Such correlations made from laboratory measurements on cores have been confirmed by full wave train acoustic logs.\(^{[13]}\)

As expected, the correlation between Poisson’s Ratio and core analysis has shown that the variations in the former are related to lithological changes. Typical values of Poisson’s Ratio for limestones and dolomites are 0.308 and 0.277 respectively. For clean sands, Poisson’s Ratio varies from as low as 0.18 for very tight sandstone to 0.27 for highly porous sand. Shaly sands exhibit still higher values of Poisson’s Ratio.

Correlation of lithology with these elastic constants is therefore obscured by variations in porosity and stress acting on the rock. However, it can be stated that limestones in general exhibit high values of elastic moduli. The values for the moduli also appear to reflect variations in the grain size of the formations (Fig. 5).

**APPLICATIONS IN WELL COMPLETION OPERATIONS**

The most important objective in drilling operations is to make a straight, true-gauge hole as economically and safely as possible. During drilling, it is often necessary to increase mud weight in order to counteract high formation pore pressures. This increase in mud weight may be a potential hazard if any of the exposed formations has low fracture pressure and consequently breaks down, causing mud loss and loss of circulation. This is a dangerous situation which cannot be remedied easily. Complete loss of circulation in some cases leads to loss of the well. Many times, such disasters could be avoided if proper techniques for calculating fracture pressure gradients had been used. The only preventive measure is to cut the mud weight or to set a protective casing. A weak formation below a casing string severely restricts mud weights during further drilling. On the other hand, no useful purpose is served in lowering a casing deeper to protect strong formations. The decision to set a casing at a particular time and depth in a well is vital for the cost and safety of operations. The mud weight program and the decision for casing a well require an accurate knowledge of expected pore pressures and fracture pressure gradients of the formations penetrated. This knowledge is a must in areas of abnormally high formation pressures.
Hubbert and Willis\textsuperscript{14} proposed that in geological regions which are tectonically relaxed and are characterized by normal faulting, the least stress will be horizontal with a magnitude of approximately one-third of the vertical stress. Assuming the ratio of horizontal to vertical stress to be a constant, however, can lead to significant errors, because it is highly variable depending upon Poisson's Ratio of the formations. Poisson's Ratio for a rock formation is a function of the rock type, porosity and shaliness.

In order to determine the magnitude of stress which resists wellbore fracturing, it is necessary to consider the forces acting on subsurface formations. In the absence of external forces, the cause of stress is the overburden and pore pressures. Under vertical stress, the rocks tend to expand laterally, but are constrained from doing so by the surrounding material. This results in a horizontal stress, which is derived from elastic theory\textsuperscript{15} as,

\[ HS = \frac{P}{1 - P} \left( P_o - P_f \right) \]  

(9)

The fracture pressure gradient is accordingly obtained as,

\[ FPG = \frac{P_f}{D} + \frac{P}{1 - P} \left( \frac{P_o - P_f}{D} \right) \]  

(10)

The data needed to compute fracture pressure gradient can be derived directly from borehole logging measurements. However, in case of difficulty in obtaining good values for shear wave travel times in friable sands, Poisson's Ratio may be empirically determined from the shaliness index in a manner similar to Equation (8). Poisson's Ratio as a function of shaliness appears reasonable since shale would act as a plastic bonding material.

This discussion has used a simplified model, where there are no additional tectonic stresses. In the generalized case, these additional stresses would be superimposed over the stresses predicted from the horizontal constraint hypothesis. Such tectonic stresses are indicated by the presence of faults, folds and domes in the area. Tectonic stresses acting in a preferred direction lead to a preferred direction for fractures.

A knowledge of fracture pressure gradient is highly desirable in the design of well completion operations such as cementing, sand consolidation, hydraulic fracturing and acidizing. In secondary recovery operations, it is necessary to remain below fracture pressures in order to avoid formation breakdown (creation of fractures) during injection. In the case of old depleted wells, formation pore pressures are usually reduced to very low values. As the fracture pressures have to partly counteract pore pressures, the fracture pressure gradients in such a case would also decrease.

**OTHER APPLICATIONS**

The number of test holes logged in mining areas has been progressively increasing. Well logs provide supplementary information which cannot be obtained easily from the analysis of recovered cores in the laboratory. In addition to the evaluation of moisture and ash content of the coal seams, well logging data have been used to determine formation strength of complete sequence of sediments in mines. Formation strength evaluation is based on the computed values of Bulk Modulus, Young's Modulus, Shear Modulus and Poisson's Ratio. Information on these elastic constants can be obtained from well logs not only for the roof and floor rocks of the mining area, but also for the entire overlying formations. Such data are useful in slope stability studies for open-pit mining as well.

The design of a subsurface mine is based on two important factors, the roof control and water restriction. For safe and economical mine design, well logging data are capable of providing important information in deciding the width and height of the mine and pillar dimensions. A combined modulus of strength computed from acoustic and density logs has been found to define the strength characteristics of the rock formations fairly well. The log data confirm the conditions known to exist in the vicinity of test holes. The combined modulus of strength correlated with actual mining experience provide a basis for the utilization of well logging methods in effective roof bolting programs. The computer processing of well log data continuously presents a quantitative analysis of coal composition, lithology and strength characteristics of the surrounding formations. Detection of shale and slate beds above coal seams is helpful in maintaining roof control in a mine. Furthermore, well logs easily identify the presence of water-bearing sands adjoining coal beds. The moisture content or water potential in the roof and floor strata can be evaluated by a complete logging suite consisting of density, acoustic, neutron, gamma ray and resistivity logs. Such a combination of logs constitutes the input data for comprehensive computer analysis. This should form the basis for the most economical and safe design of mining operations. Collaboration between logging and mining industries will hopefully lead to further improvements in these techniques.

**CONCLUSIONS**

Well logging methods are available for the measurement of acoustic velocities of compressional
and shear waves in rock formations. A combination of acoustic transit time with bulk density data provides a means of estimation of in-situ dynamic elastic properties of rocks. Elastic constants computed from well log data can be better correlated to the mechanical behavior of formations than those determined from laboratory tests. The Sidewall Acoustilog® improves identification of shear wave arrivals.

Shear Modulus plays a key role in the prediction of sanding problems in friable sands. A combination modulus of strength or the product of Bulk and Shear Moduli can be used to denote intrinsic strength of sand formations. Maximum safe flow rate in a well can be estimated from well logs if production and well logging data of other wells in the area are available. Poisson's Ratio is related to lithology and shale content of the sedimentary rocks. Elastic constants obtained from well logs contribute toward better design of operations during drilling, completion, stimulation and secondary recovery. Identification of beds susceptible to fracturing and estimation of fracture pressure gradients are vital in the planning and execution of efficient well completions. In the mining industry, a knowledge of formation strength characteristics helps in the design of installation of mining facilities.

**NOMENCLATURE**

\[
\begin{align*}
Y &= \text{Young's Modulus, psi} \\
B &= \text{Bulk Modulus, psi} \\
S &= \text{Shear Modulus, psi} \\
P &= \text{Poisson's Ratio} \\
\rho_b &= \text{Bulk density, g/cm}^3 \\
\Delta t_c &= \text{Compressional wave specific acoustic time, \(\mu\text{sec/ft}\)} \\
\Delta t_s &= \text{Shear wave specific acoustic time, \(\mu\text{sec/ft}\)} \\
K &= \text{Combined modulus of strength, psi} \\
q &= \text{Shaliness index} \\
\phi_{AC} &= \text{Apparent acoustic log porosity} \\
\phi_d &= \text{Apparent density log porosity} \\
HS &= \text{Horizontal stress, psi} \\
P_o &= \text{Overburden pressure, psi} \\
P_f &= \text{Pore pressure, psi} \\
FPG &= \text{Fracture pressure gradient, psi/ft} \\
D &= \text{Depth, ft}
\end{align*}
\]

**REFERENCES**


CORRELATION BETWEEN STATIC AND DYNAMIC BULK MODULI

![Graph showing correlation between static and dynamic bulk moduli.](image)

**FIGURE 1**
Correlation between static and dynamic Bulk Moduli. (After Ref. 3)

SIDEWALL ACOUSTILOG®

![Sidewall acoustic compressional and shear wave logs.](image)

**FIGURE 2**
Sidewall acoustic compressional and shear wave logs.
FIGURE 3
Variable Density Display/Signature Log in varying lithology.

FIGURE 4
Compressional wave transit time vs shear wave transit time based on laboratory measurements. (After Ref. 13)

FIGURE 5
Elastic constants vs. Lithology. (After Ref. 4)